Answer ALL TWENTY questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

1 (a) Simplify $e^8 \div e^2$

e^{8-2}

e^6

(1)

(b) Expand and simplify $(x - 3)(x + 1)$

$x \times x = x^2$

$x \times 1 = 1x$

$-3 \times x = -3x$

$-3 \times 1 = -3$

$x^2 - 2x - 3$

(2)

(Total for Question 1 is 3 marks)

---

2 Here is a right-angled triangle.

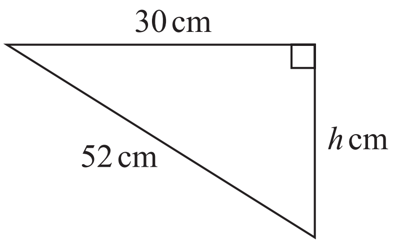


Diagram **NOT** accurately drawn

Calculate the value of h.
Give your answer correct to 3 significant figures.

$h = \sqrt{52^2 - 30^2}$

$= 42.473...$

$\uparrow$

$h =$ 42.5

(Total for Question 2 is 3 marks)

3 There are 54 fish in a tank.
Some of the fish are white and the rest of the fish are red.

Jeevan takes at random a fish from the tank.
The probability that he takes a white fish is $\frac{4}{9}$

(a) Work out the number of white fish originally in the tank.

54

W $\frac{4}{9}$

R $\frac{5}{9}$

$$\overset{\times 6}{\underset{\times 6}{\frac{4}{9} \quad \frac{24}{54}}}$$

24

(2)

Jeevan puts the fish he took out, back into the tank.
He puts some more white fish into the tank.

Jeevan takes at random a fish from the tank.
The probability that he takes a white fish is now $\frac{1}{2}$

(b) Work out the number of white fish Jeevan put into the tank.

W 24

R 30

$P(W) = \frac{1}{2}$ so white = 30

$30 - 24 = 6$

6

(2)

(Total for Question 3 is 4 marks)

DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA

4 The diagram shows the front of a wooden door with a semicircular glass window.

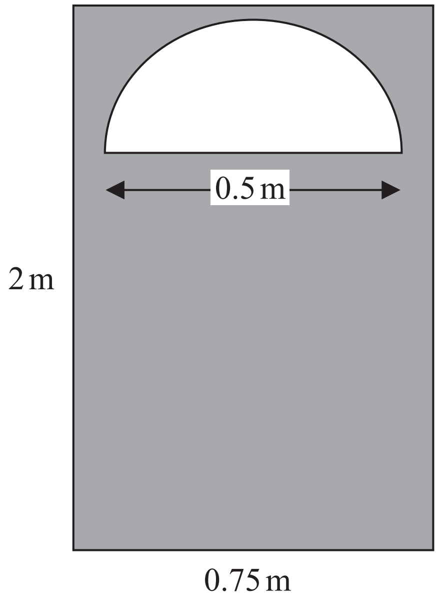


Diagram **NOT** accurately drawn

d = 0.5
r = 0.25

⌓ area
$= \frac{1}{2} \pi \times 0.25^2$
= 0.098

Julie wants to apply 2 coats of wood varnish to the front of the door, shown shaded in the diagram.

250 millilitres of wood varnish covers 4 m^2 of the wood.

Work out how many millilitres of wood varnish Julie will need.
Give your answer correct to the nearest millilitre.

diameter = 0.5 m   radius = 0.25

area = $\frac{1}{2} \times \pi \times 0.25^2$ = 0.098...

Door => 2 × 0.75 − 0.098...

1.5 − 0.098

= 1.401.. m^2

2 coats = 2.80365.. m^2

Vanish   250ml = 4 m^2) ÷ 4
62.5ml = 1 m^2) × 2.80...
175.2... = 2.80...
↑
(ml)

175 millilitres

(Total for Question 4 is 5 marks)

5 Yasmin has some identical rectangular tiles.
Each tile is *L* cm by *W* cm.

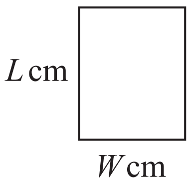


Diagram **NOT** accurately drawn

Using 9 of her tiles, Yasmin makes rectangle *ABCD*, shown in the diagram below.

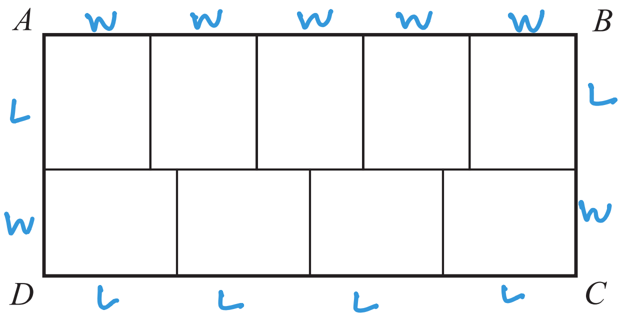


Diagram **NOT** accurately drawn

The area of *ABCD* is 1620 cm^2

Work out the value of *L* and the value of *W*.

$5W = 4L \quad \text{so} \quad L = \frac{5}{4}W$

area $\quad 5W \times (L + W) = 1620$

$5W \times \left(\frac{5}{4}W + w\right) = 1620$

$\frac{25W^2}{4} + 5W^2 = 1620$

$11.25\,W^2 = 1620$

$W = \sqrt{\frac{1620}{11.25}} = 12$

using $\quad 5W = 4L$

$5 \times 12 = 4L$

$\therefore \quad L = \frac{60}{4} = 15$

L = 15 *W* = 12

(Total for Question 5 is 5 marks)

6 Alison buys 5 apples and 3 pears for a total cost of \$1.96
Greg buys 3 apples and 2 pears for a total cost of \$1.22

Michael buys 10 apples and 10 pears.

Work out how much Michael pays for his 10 apples and 10 pears.
Show your working clearly.

① $5A + 3P = 1.96 \times 2$

② $3A + 2P = 1.22 \times 3$

③ $10A + 6P = 3.92$

④ $9A + 6P = 3.66$

$A = 0.26$

$3 \times 0.26 + 2P = 1.22$

$2P = 1.22 - 0.78$

$P = 0.22$

Michael 10×0.26 $2.60 + 2.20$

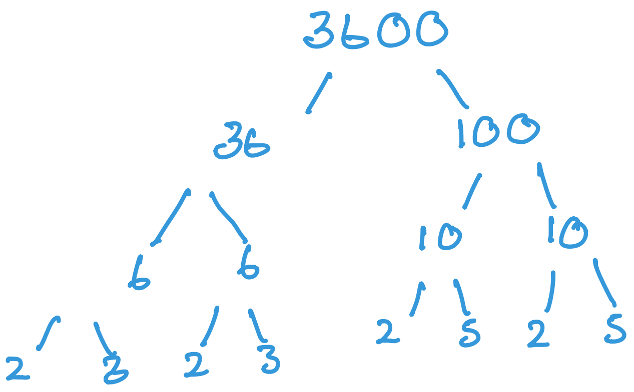
$+ 10 \times 0.22$

\$ 4.80

(Total for Question 6 is 5 marks)

---

7 Write 3.6×10^3 as a product of powers of its prime factors.
Show your working clearly.



$2^4 \times 3^2 \times 5^2$

$2^4 \times 3^2 \times 5^2$

(Total for Question 7 is 3 marks)

8 In 2018, the population of Sydney was 5.48 million.
This was 22% of the total population of Australia.

Work out the total population of Australia in 2018
Give your answer correct to 3 significant figures.

5·48 m = 22% ÷22

= 1% × 100

24·909.. = 100%

(3sf.)

24·9 million

(Total for Question 8 is 3 marks)

---

9 (i) Solve the inequalities $-7 \leqslant 2x - 3 < 5$

+3 +3 +3

$-4 \leqslant 2x < 8$

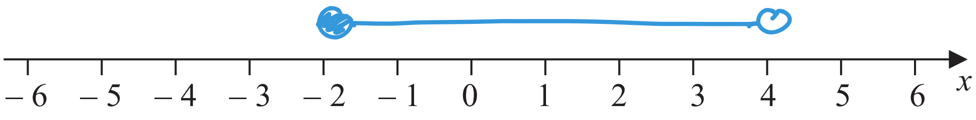
÷2 ÷2 ÷2

$-2 \leqslant x < 4$

$-2 \leqslant x < 4$

(3)

(ii) On the number line, represent the solution set to part (i)



(2)

(Total for Question 9 is 5 marks)

10 A solid aluminium cylinder has radius 10 cm and height h cm.

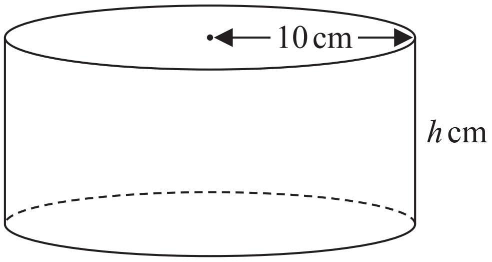


Diagram **NOT** accurately drawn

The mass of the cylinder is 5.4 kg.
The density of aluminium is 0.0027 kg/cm^3

$D = \frac{m}{V}$

Calculate the value of h.
Give your answer correct to one decimal place.

$V = \frac{m}{D}$

Volume $= \frac{5.4}{0.0027} = 2000 \text{ cm}^3$

Volume is also $\Rightarrow \pi \times 10^2 \times h$

so

$100\pi h = 2000$

$h = \frac{2000}{100\pi}$

$= 6.3661\ldots$

$\uparrow$

(1dp)

$h =$ 6.4

(Total for Question 10 is 5 marks)

11 The table gives information about the times taken by 90 runners to complete a 10 km race.

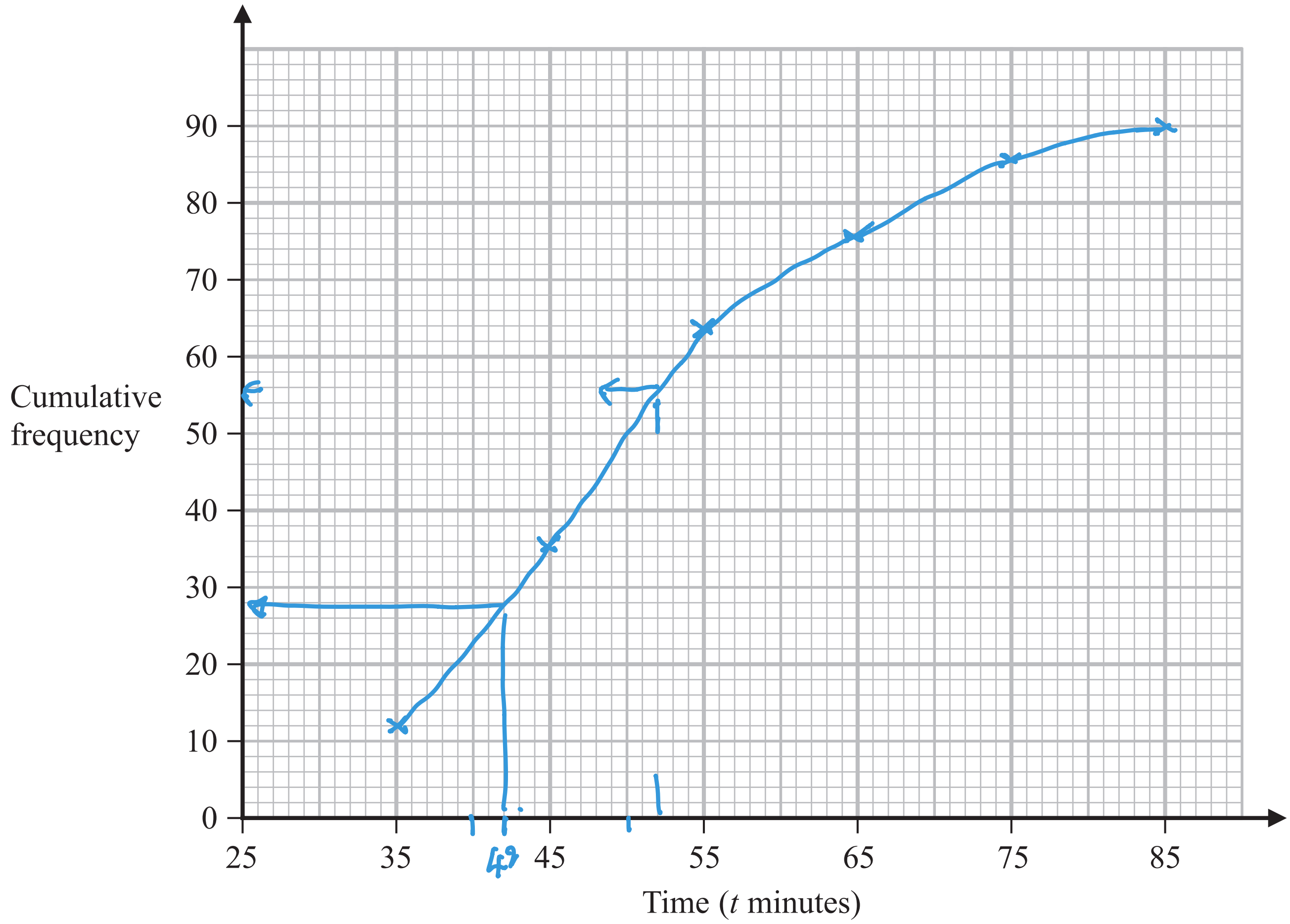
Time (t minutes)	**Frequency**
$25 < t \leqslant 35$	12
$35 < t \leqslant 45$	24
$45 < t \leqslant 55$	28
$55 < t \leqslant 65$	12
$65 < t \leqslant 75$	10
$75 < t \leqslant 85$	4

(a) Complete the cumulative frequency table.

Time (t minutes)	**Cumulative frequency**
$25 < t \leqslant 35$	12
$25 < t \leqslant 45$	36
$25 < t \leqslant 55$	64
$25 < t \leqslant 65$	76
$25 < t \leqslant 75$	86
$25 < t \leqslant 85$	90

(1)

(b) On the grid below, draw a cumulative frequency graph for your table.



(2)

28 56

Any runner who completed the race in a time T minutes such that $42 < T \leqslant 52$ minutes was awarded a silver medal.

(c) Use your graph to find an estimate for the number of runners who were awarded a silver medal.

56 - 28

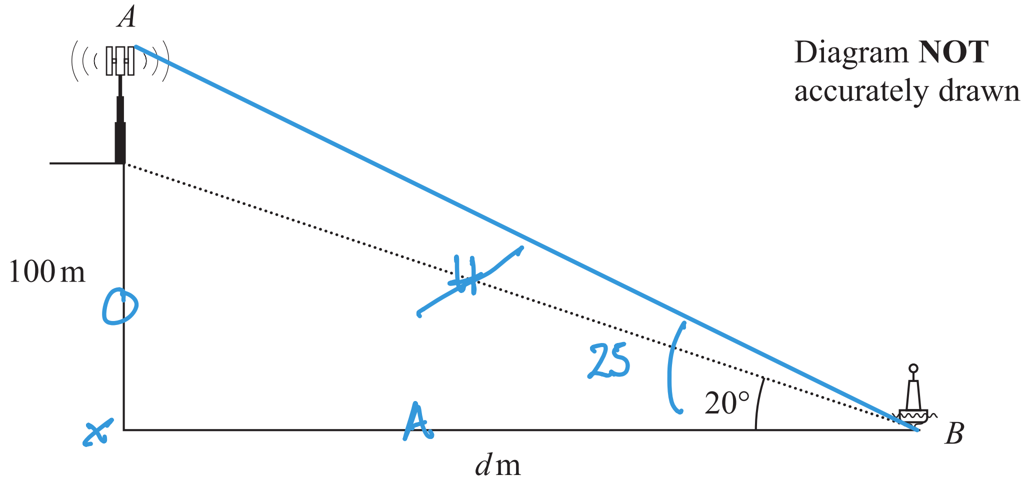
28 runners

(2)

(answer between 25 and 29 accepted dependents on your graph!)

(Total for Question 11 is 5 marks)

12 The diagram shows a vertical cliff with a vertical radio mast on top of the cliff and a buoy in the sea.



The height of the cliff is 100 metres.
The buoy is at the point *B* that is *d* metres from the base of the cliff.
The angle of elevation from *B* to the top of the cliff is 20°

(a) Calculate the value of *d*.
Give your answer correct to 3 significant figures.

$\tan 20 = \frac{100}{d}$

$d = \frac{100}{\tan 20}$

$= 274.747...$

(3sf)

d = 275

(3)

The point *A* at the top of the radio mast is vertically above the top of the cliff.
The angle of elevation from *B* to *A* is 25°

(b) Calculate the height of the radio mast.
Give your answer correct to 3 significant figures.

$\tan 25 = \frac{XA}{274.7...}$

$XA = 274.7.. \times \tan 25$

$= 128.1164...$

so height of mast $= 128.11.. - 100$

$= 28.1164..$

(3sf.)

28.1 m

(3)

(Total for Question 12 is 6 marks)

13 Here is a triangle *XYZ*.

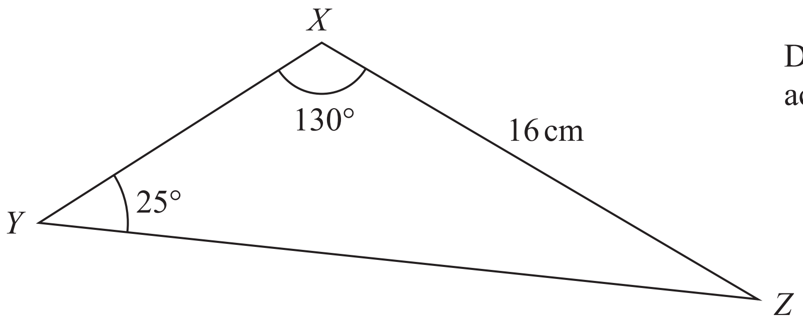


Diagram **NOT** accurately drawn

The length *XZ* and the angles *YXZ* and *XYZ* are each given correct to 2 significant figures.

Calculate the upper bound for the length *YZ*.
Give your answer correct to one decimal place.
Show your working clearly.

$$\begin{tikzcd} & 25.5 \\ 25 \arrow[ur] \arrow[dr, dash] & \\ & 24.5 \end{tikzcd}$$

$$\begin{tikzcd} & 135 \\ 130 \arrow[ur] \arrow[dr] & \\ & 125 \\ 120 & \end{tikzcd}$$

$$\begin{tikzcd} & 16.5 \\ 16 \arrow[ur] \arrow[dr] & \\ & 15.5 \end{tikzcd}$$

$$\frac{16.5}{\sin 24.5} = \frac{YZ}{\sin 125}$$

$$YZ = \frac{16.5}{\sin 24.5} \times \sin 125$$

$$= \underset{\overset{\uparrow}{(1dp)}}{32.5927\ldots}$$

32.6 cm

(Total for Question 13 is 3 marks)

14 *ABCDEF* and *GHIJKL* are regular hexagons each with centre *O*.

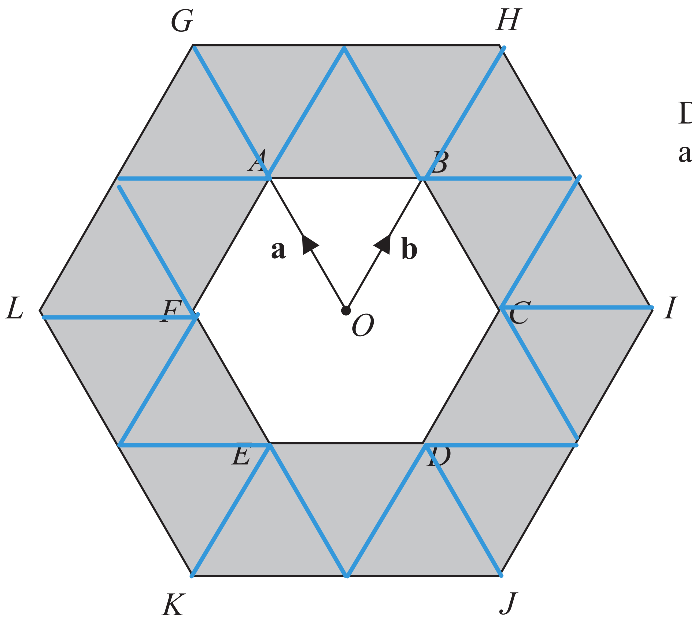


Diagram **NOT** accurately drawn

GHIJKL is an enlargement of *ABCDEF*, with centre *O* and scale factor 2

$\overrightarrow{OA} = \mathbf{a}$ $\overrightarrow{OB} = \mathbf{b}$

(a) Write the following vectors, in terms of **a** and **b**.
Simplify your answers.

(i) $\overrightarrow{AB}$

$b - a$

(1)

(ii) $\overrightarrow{KI}$

$2(b-a) + 2b$

$= 2b - 2a + 2b$

$4b - 2a$

(2)

(iii) $\overrightarrow{LD}$

$2(b-a) - a$

$= 2b - 3a$

$2b - 3a$

(2)

The triangle *OAB* has an area of 5 cm^2

(b) Calculate the area of the shaded region.

18×5

90 cm^2

(3)

(Total for Question 14 is 8 marks)

/

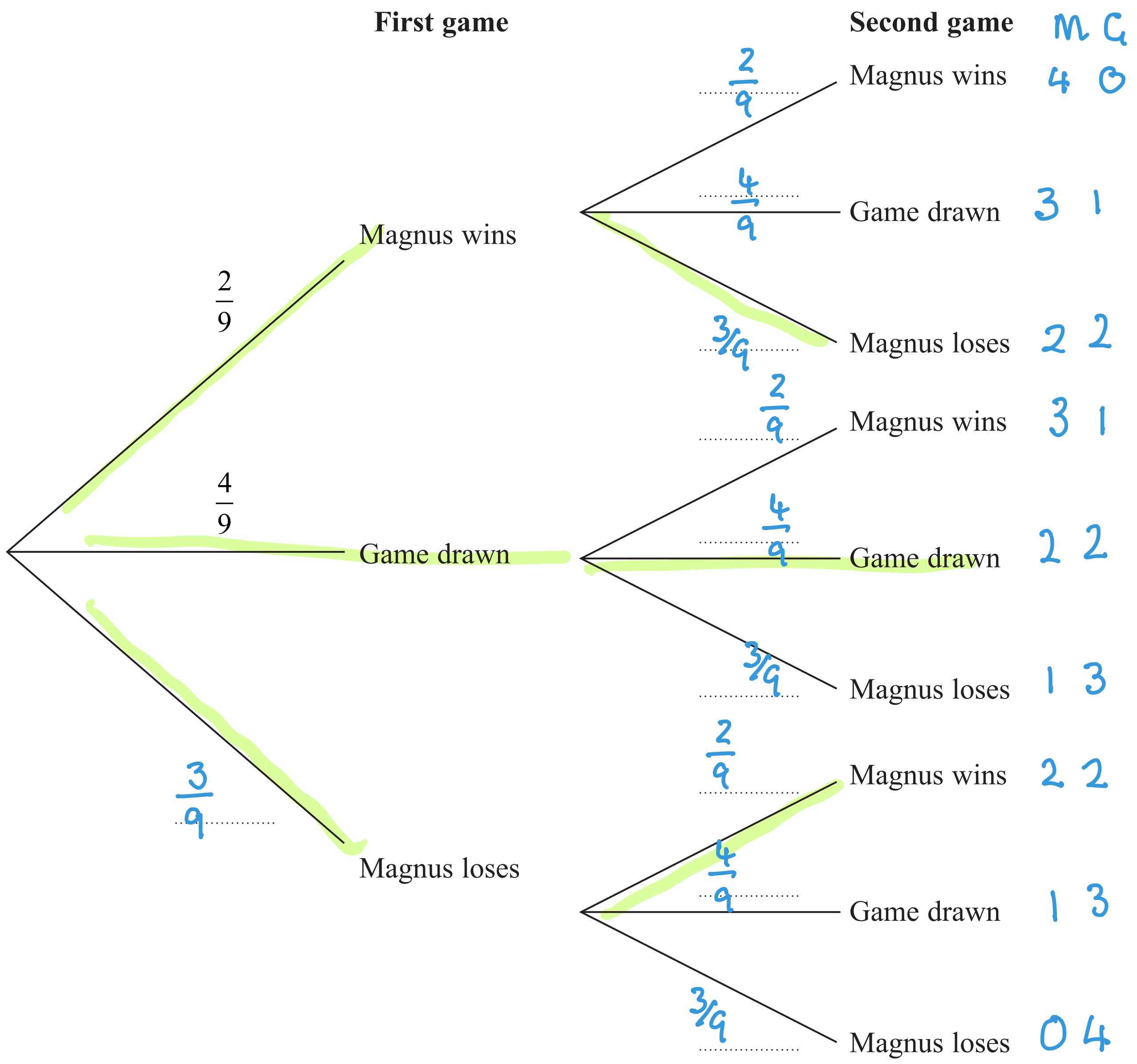
15 Magnus and Garry play 2 games of chess against each other.

The probability that Magnus beats Garry in any game is $\frac{2}{9}$

The probability that any game between Magnus and Garry is drawn is $\frac{4}{9}$

The result of any game is independent of the result of any other game.

(a) Complete the probability tree diagram.



(2)

W:2 L:0 D:1

For each game of chess,

the winner gets 2 points and the loser gets 0 points,
when the game is drawn, each player gets 1 point.

(b) Work out the probability that, after 2 games, Magnus and Garry have the same number of points.

$$\frac{2}{9} \times \frac{3}{9} + \frac{4}{9} \times \frac{4}{9} + \frac{3}{9} \times \frac{2}{9}$$

$$= \frac{6}{81} + \frac{16}{81} + \frac{6}{81}$$

$$= \frac{28}{81}$$

$\frac{28}{81}$

(3)

Magnus and Garry now play a third game of chess.

(c) Work out the probability that, after 3 games, Magnus and Garry have the same number of points.

m G

W L D 3 3 x6

D D D 3 3

$$6 \times \left(\frac{2}{9} \times \frac{3}{9} \times \frac{4}{9} \right)$$

$$+ \left(\frac{4}{9} \times \frac{4}{9} \times \frac{4}{9} \right)$$

$$= \frac{144}{729} + \frac{64}{729}$$

$\frac{208}{729}$

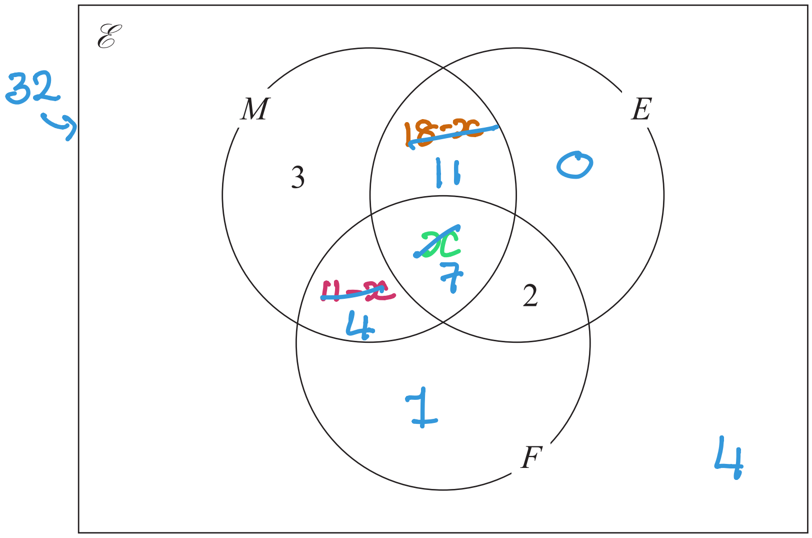
(3)

(Total for Question 15 is 8 marks)

16 There are 32 students in a class.
In one term these 32 students each took a test in Maths (M), in English (E) and in French (F).

25 students passed the test in Maths.
20 students passed the test in English.
14 students passed the test in French.
✓18 students passed the tests in Maths and English.
✓11 students passed the tests in Maths and French.
✓ 4 students failed all three tests.
✓ x students passed all three tests.

The incomplete Venn diagram gives some more information about the results of the 32 students.



E
$= 20 - (11+7+2$
$= 0$

F
$14 - (4+7+2)$

(a) Use all the given information about the results of students who passed the test in Maths to find the value of x.

$$25 = 3 + 18 \cancel{-x} \cancel{+x} + 11 - x$$

$$25 = 32 - x$$

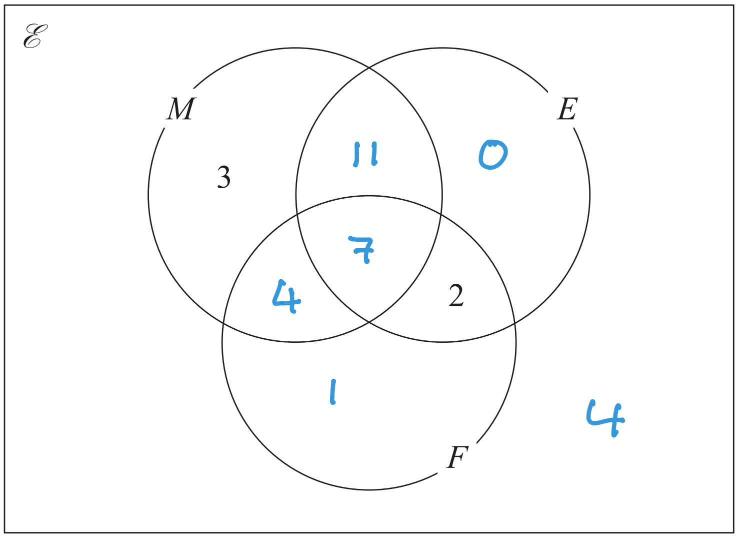
$$x = 32 - 25$$

$$= 7$$

$x =$ 7

(2)

(b) Use your value of x to complete the Venn diagram to show the number of students in each subset.



(2)

25

A student who passed the test in Maths is chosen at random.

(c) Find the probability that this student failed the test in French.

$\frac{14}{25}$

(1)

(Total for Question 16 is 5 marks)

---

17 (a) Factorise $6y^2 - y - 5$

$6 \times 5 = 30$

1, 30
2, 15
3, 10
$\boxed{5, 6}$

$6y^2 - 6y + 5y - 5$

$6y(y-1) + 5(y-1)$

$(6y+5)(y-1)$

$(6y+5)(y-1)$

(2)

(b) Make f the subject of $w = \dfrac{2f + 3}{8 - f}$

$w(8 - f) = 2f + 3$

$8w - fw = 2f + 3$

$8w - 3 = 2f + fw$

$8w - 3 = f(2 + w)$

$f = \dfrac{8w - 3}{2 + w}$

$f = \dfrac{8w-3}{2+w}$

(3)

(c) Express $4x^2 - 8x + 7$ in the form $a(x + b)^2 + c$ where a, b and c are integers.

$4(x^2 - 2x) + 7$

$= 4[(x - 1)^2 - 1] + 7$

$= 4(x - 1)^2 - 4 + 7$

$= 4(x - 1)^2 + 3$

$4(x-1)^2 + 3$

(3)

(Total for Question 17 is 8 marks)

18 $0.4\dot{x}$ is a recurring decimal.
x is a whole number such that $1 \leqslant x \leqslant 9$

$100y = 4 \cdot xxxx \ldots$
$y = 0 \cdot 4xxx \ldots$

Find, in terms of x, the recurring decimal $0.4\dot{x}$ as a fraction.
Give your fraction in its simplest form.
Show clear algebraic working.

$y = 0 \cdot 4xx \ldots$

$10y = 4 \cdot xxxx$
$100y = 4x \cdot xxxx$

$100y - 10y$
$= 90y$

$40 + (x - 4)$

$90y = 40 + x - 4$

$y = \dfrac{36 + x}{90}$

$\dfrac{36 + x}{90}$

(Total for Question 18 is 3 marks)

19 *ABCED* is a five-sided shape.

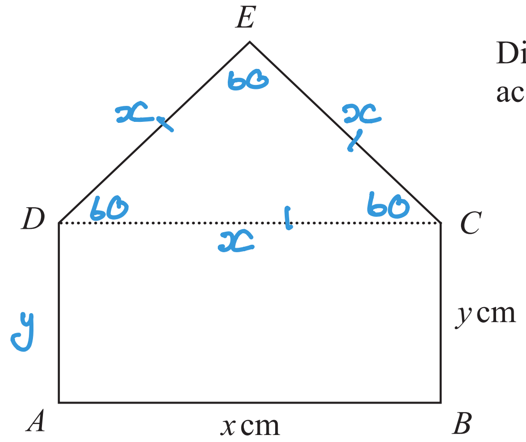


Diagram **NOT** accurately drawn

DE = EC = DC

ABCD is a rectangle.
CED is an equilateral triangle.

$AB = x$ cm $BC = y$ cm

The perimeter of *ABCED* is 100 cm.
The area of *ABCED* is R cm^2

(a) Show that $R = \frac{x}{4}\left(200 - \left[6 - \sqrt{3}\right]x\right)$

Perimeter $100 = 3x + 2y$ $y = \frac{100 - 3x}{2}$

Area Δ EDC $= \frac{1}{2} \times x \times x \times \sin 60 = \frac{\sqrt{3}x^2}{4}$

Area ABCD $= xy = x\left(\frac{100 - 3x}{2}\right)$

Total area $= R = x\left(\frac{100 - 3x}{2}\right) + \frac{\sqrt{3}}{4}x^2$

$= x\left(\frac{200 - 6x}{4}\right) + \frac{\sqrt{3}}{4}x^2$

$= \frac{x}{4}(200 - 6x + \sqrt{3}x)$

$= \frac{x}{4}(200 - x(6 - \sqrt{3}))$ as required.

(3)

(b) (i) Find the value of x for which R has its maximum value.

Give your answer in the form $\dfrac{p}{q - \sqrt{3}}$ where p and q are integers.

$$R = \frac{x}{4}\left(200 - (6 - \sqrt{3})x\right)$$

$$= \frac{200x}{4} - \frac{\overset{3}{\cancel{6}}x^2}{2\cancel{4}} + \frac{\sqrt{3}x^2}{4}$$

$$\frac{dR}{dx} = 50 - \cancel{2} \times \frac{3}{\cancel{2}} \times x + \frac{\cancel{2}\sqrt{3}}{\cancel{4}2}x = 0$$

$\times 2$

$$100 - 6x + \sqrt{3}x = 0$$

$$100 - x(6 + \sqrt{3}) = 0$$

$$100 = x(6 - \sqrt{3})$$

$$x = \frac{100}{6 - \sqrt{3}}$$

$x =$ $\dfrac{100}{6 - \sqrt{3}}$

(2)

(ii) Explain why the maximum value of R is given by this value of x.

R is a quadratic with a negative coefficient of x^2

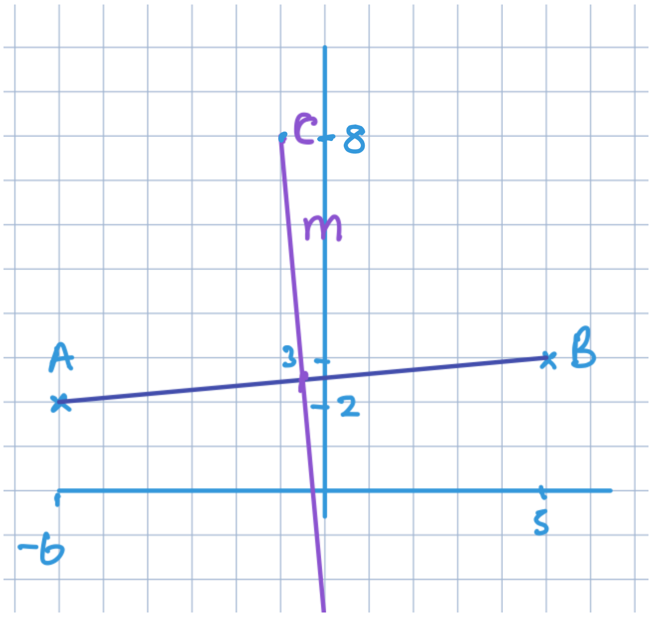
(1)

(Total for Question 19 is 6 marks)

Turn over for Question 20

20 The straight line **L** passes through point *A* (– 6, 2) and point *B* (5, 3)
The straight line **M** is perpendicular to **L** and passes through the midpoint of *A* and *B*.
The line **M** intersects the line $x = -1$ at point *C*.

Calculate the area of triangle *ABC*.



midpoint AB

$\frac{-6+5}{2}, \frac{2+3}{2} = (-0.5, 2.5)$

gradient AB

$= \frac{1}{11}$

so gradient of M $= -11$

equation of M $y = -11x + c$ $\quad (-0.5, 2.5)$ $\quad x \quad y$

$2.5 = -11 \times -0.5 + c$

$2.5 = 5.5 + c$

$c = -3$ $\qquad$ so $y = -11x - 3$

intersects at C $\quad x = -1$

$y = -11 \times -1 - 3 = 8$

Area ABC $= \frac{1}{2} \times \sqrt{11^2 + 1^2} \times \sqrt{0.5^2 + 5.5^2}$

$= \frac{1}{2} \times \sqrt{122} \times \frac{\sqrt{122}}{2}$

$= 30.5$

30.5

(Total for Question 20 is 7 marks)

TOTAL FOR PAPER IS 100 MARKS